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Mapping Lorentz Invariance Violations into Equivalence Principle Violations

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Abstract

We point out that equivalence principle violations, while not dynamically equivalent, produce the same kinematical effects as Lorentz invariance violations for particle processes in a constant gravitational potential. This allows us to translate many experimental bounds on Lorentz invariance violations into bounds on equivalence principle violations. The most stringent bound suggests that a postive signal in an Eötvös experiment may be at least seven orders of magnitude beyond current technology.

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Recently, new limits have been placed on possible violations of Lorentz invariance (LIV) [1], and it has been noted that such a violation can turn on (turn off) processes at sufficiently high energy that are otherwise kinematically forbidden (allowed) [1,2]. In particular, it has been suggested that such a violation may provide a satisfactory answer to the observation of cosmic ray protons at energies above the GZK cutoff [3,4]. In this paper we demonstrate that all effects of the type of special relativity violation entertained by Coleman and Glashow can, for all practical purposes in our solar system, be obtained by the less rash hypothesis of small violations of the equivalence principle (EPV) through a lack of universality in gravitational coupling strengths to matter. This has previously been noted in the case of neutrino oscillations [5] but not for other phenomena. With this equivalence established, we use the same phenomena to place limits on gravitational couplings to ordinary matter and set goals for future Eötvös type experiments.

The proposal of Coleman and Glashow [1,6] is to modify the the usual free particle dispersion relation of special relativity in a minimal way by allowing the world line of each species to be determined by its own limiting velocity , i.e.

$$E^2 = (pc_a)^2 + (mc_a^2)^2 \tag{1}$$

The limits on values of c_a that have been obtained as well as suppression of the GZK bound arise from the fact that in this scheme some particle velocities may exceed the limiting velocities of other particles.

The same kind of "superluminal" effects happen if universality of the gravitational coupling is relaxed without giving up special relativity. This is because current observations are made in a medium containing an essentially constant gravitational potential, which does nothing more than change the limiting speeds of particles in the medium in a species dependent manner [7]. Since we are concerned only with kinematics, we shall treat all particles as spinless and therefore described by a Klein-Gordan equation. The effect of gravity is to modify the metric from flat space time, $\eta_{\alpha\beta}$, to the metric $\eta_{\alpha\beta} + h_{\alpha\beta}$. In the weak field approximation to Einstein gravity written in the harmonic gauge, $h_{\alpha\beta}$ is determined by [8]

$$\partial_\alpha \partial^\alpha h_{\mu\nu} = -16\pi G S_{\mu\nu} \tag{2}$$

where G is Newton's constant and the tensor source current, $S_{\mu\nu}$, is related to the energy momentum tensor, $T_{\mu\nu}$, by

$$S_{\mu\nu} = T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} T^\lambda_\lambda \tag{3}$$

For a static matter source distribution producing Newtonian potential Φ, the solution for $h_{\mu\nu}$ is [9]

$$h_{\mu\nu} = 2\Phi\delta_{\mu\nu} \tag{4}$$

We introduce a breakdown of Einstein gravity by allowing the coupling of the metric to the Klein-Gordan equation for particle species a to be nonuniversal through a parameter f_a, which is unity in Einstein theory, i.e., we make the replacement

$$h_{\mu\nu} \rightarrow f_a h_{\mu\nu} \tag{5}$$

In the case of a constant Newtonian potential, the otherwise free particle energy-momentum relation thus becomes

$$(1 + 2f_a\Phi)E^2 = (1 - 2f_a\Phi)p^2 + m_o^2 \tag{6}$$

where m_o is the mass in a true vacuum. If all f_a were equal, this constant potential would have no physical significance, because it would just be absorbed in a redefinition of the velocity of light. However, if two particles have unequal values of f_a, a constant value of the Newtonian potential acquires physical significance, since it means that their limiting velocities will differ.

At some level, the physical significance of the absolute value of the Newtonian potential will have to show up in a modification of Eq. (2), since it is invariant under $\Phi \to \Phi+$constant. We would anticipate that the EPV portions of the coupling will contribute to the graviton self energy to produce a small but non-zero graviton mass. Of course, this is a matter of conjecture, since we have absolutely no way to calculate such an effect. The reader may wonder if such a graviton mass is ruled out by the observation of Van Dam and Veltman [10] that a graviton with any mass is inconsistent with the experimental accuracy on observations of the advance of the perihelion of Mercury. We would argue that such an inference is not possible, because generating a graviton mass in this way could very well result in a smooth transition from the massive to the massless graviton similar, for example, to that devised by Schwinger [11].

In order to cast the LIV and EPV dispersion relations in the same form, we simply note that in the latter case it is natural to redefine the mass such that in the low energy limit the kinetic energy is still related to the observed mass m by the usual nonrelativistic relation $K.E. = m^2/2E$, in which case we have

$$m^2 = \frac{(1 + 2f_a\Phi)}{(1 - 2f_a\Phi)^2} m_o^2 \tag{7}$$

and

$$E^2 = (pc_a)^2 + (mc_a^2)^2 \tag{8}$$

where c_a and f_a are related by

$$c_a = \frac{(1 - 2f_a\Phi)}{(1 + 2f_a\Phi)} \simeq 1 - 4f_a\Phi. \tag{9}$$

The kinematical equivalence of LIV and EPV is now manifest, and we are now in a position to translate all of the limits on differences of various c_a compiled by Coleman and Glashow (CG) into limits on differences of f_a values once we have a value for the Newtonian potential. From the compilation of [12], we see that for experiments in our galaxy the dominant contribution observed thus arises from the great attractor and has a value of $\Phi = -3 \times 10^{-5}$ [13]. Assuming that any induced graviton mass as discussed above is irrelevant at this distance scale, we use this value as a lower limit (there may be other bodies out there that give an even larger contribution) for $|\Phi|$. In Table 1 we give the CG limits on limiting velocities and the corresponding limits on the gravitation parameters.

It should be emphasized that while the LIV and EPV scenarios are kinematically indistinguishable, they can be distinguished by the dynamics of low energy experiments, because there is no trace of LIV in the low energy form of the kinetic energy. For example, in an Eötvös experiment, even if LIV is present, there will be no positive signal unless the equivalence principle is violated. The limit on f_p obtained from ultra-high energy cosmic rays suggests that observation of a positive signal in any Eötvös experiment may be at least some seven orders of magnitude away from current technology.

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TABLES

Related experiment	Bound on LIV	Bound on EPV
observation of ultra high energy cosmic rays [6]	$c_p - c_\gamma < 1 \times 10^{-23}$	$f_p - f_\gamma < 1 \times 10^{-19}$
high precision spectroscopic experiments that fail to find anisotropies in CMB [14]	$\lvert c_m - c_\gamma \rvert < 6 \times 10^{-22}$	$\lvert f_m - f_\gamma \rvert < 6 \times 10^{-18}$
search for high energy neutrino oscillation [15]	$\lvert c' - c \rvert_{\nu_e \nu_\mu} < 6 \times 10^{-22}$	$\lvert f' - f \rvert_{\nu_e \nu_\mu} < 6 \times 10^{-18}$
muon fluxes in the air shower [16]	$\lvert c' - c \rvert_{\mu e} < 4 \times 10^{-21}$	$\lvert f' - f \rvert_{\mu e} < 4 \times 10^{-17}$
energy dependence of K_L-K_S mass difference [17]	$\lvert c_{K_L} - c_{K_S} \rvert < 3 \times 10^{-21}$	$\lvert f_{K_L} - f_{K_S} \rvert < 3 \times 10^{-17}$

TABLE I. Bounds on the equivalence principle violation parameters mapped from bounds on the Lorentz invariance violation parameters